TMSP 21/22
Problems 2, 22.03.2022
(Please solve each problem on a separate sheet of paper)

1. The Widom scaling hypothesis predicts the following scaling form of the singular part of the free energy density:

$$g(\tau, h) = |\tau|^{2-\alpha}\, \Phi_{\pm}\left(\frac{h}{|\tau|^{\beta\delta}}\right) \quad . \tag{1}$$

Consider the Landau theory in the presence of uniform magnetic field h

$$\beta\mathcal{H}_{eff}(m) = \beta G_0(T) + V\left(\frac{a\tau}{2}m^2 + \frac{u}{4!}m^4 - hm\right) \tag{2}$$

and show that within Landau theory one indeed obtains the above form of $g(\tau, h)$. What can one say about the form of functions $\Phi_{\pm}(x)$?

2. Consider the Landau theory and calculate $c_H(T, h = 0)$. What is the corresponding value of the critical exponent α ?

3. **Molecular field theory.**

1. consider $h = 0$ and solve the corresponding equation for $m(\tau, 0)$ for $|\tau| \ll 1$. What is the corresponding value of exponent β?
2. evaluate exponent δ.
3. evaluate and make schematic plot of $g(\tau, h = 0)$
4. evaluate $\Delta c_H = c_H(T_c^+, 0) - c_H(T_c^-, 0)$. What is the value of exponent α?

4 (not obligatory). **Potts model**
Consider lattice with N sites. Each site is occupied by a "spin" σ which can take q-values such that $\sigma_i = 1, \ldots, q$. The Potts Hamiltonian has the form

$$\mathcal{H}_{Potts}(\{\sigma_i\}) = -J \sum_{<ij>} \delta_{\sigma_i,\sigma_j} - h \sum_{i}^{N} \delta_{\sigma_i,1} \tag{3}$$

where $\delta_{a,b}$ is the Kronecker delta. Impose periodic boundary conditions and - with the help of transfer matrix method - evaluate the partition function $Z(T, h, N)$ and $g(T, h)$.